5/24/17 "Dont find fault, find a remedy" -Henry Ford

HW: "Independent Events" homework section

AIM: What does it mean to be Independent?

Warm Up:

CONDITIONAL PROBABILITY
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Given that $P(\text{B} \mid \text{A})$ means the probability of event B occurring given that event A will occur or has occurred, which of the following correctly calculates this probability?

(1) $\frac{P(\text{B})}{P(\text{A})}$ (3) $\frac{P(\text{A})}{P(\text{B})}$

(2) $\frac{P(\text{A and B})}{P(\text{B})}$ (4) $\frac{P(\text{A and B})}{P(\text{A})}$

(4)

APPLICATIONS

2. Of the 650 juniors at Arlington High School, 468 are enrolled in Algebra II, 292 are enrolled in Physics, and 180 are taking both courses at the same time. If one of the 650 juniors was picked at random, what is the probability they are taking Physics, if we know they are in Algebra II?

(1) 0.38 (3) 0.45

(2) 0.62 (4) 0.58

$$P(\text{P}|\text{A2}) = \frac{n(\text{P and A2})}{n(\text{A2})} = \frac{180}{468} = 0.38$$

(1)

3. Historically, a given day at the beginning of March in upstate New York has a 18% chance of snow and a 12% chance of rain. If there is a 4% chance it will rain and snow on a day, then calculate each of the following:

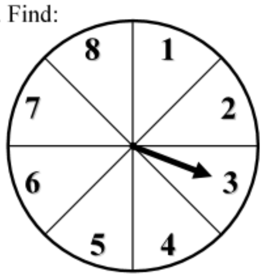
(a) the probability it will rain given that it is snowing, i.e.

$P(\text{rain} \mid \text{snow}) = \frac{P(\text{R and S})}{P(\text{S})} = \frac{0.04}{0.18} \approx 0.22$

(b) the probability it will snow given that it is raining, i.e.

$P(\text{snow} \mid \text{rain}) = \frac{P(\text{R and S})}{P(\text{R})} = \frac{0.04}{0.12} \approx 0.33$

4. A spinner is spun around a circle that is divided up into eight equally sized sectors. Find:



(a) $P(\text{perfect square} \mid \text{even})$

$= \frac{n(\text{S and E})}{n(\text{E})} = \frac{1}{4}$

(b) $P(\text{odd} \mid \text{prime})$

$= \frac{n(\text{O and P})}{n(\text{P})} = \frac{3}{4}$

(c) What is more likely: getting a multiple of four given we spun an even or getting an odd, given we spun a number greater than 2? Support your answer.

$P(\text{M4}|\text{E}) = \frac{n(\text{M4 and E})}{n(\text{E})} = \frac{2}{4} = \frac{1}{2}$

$P(\text{O}|{>2}) = \frac{n(\text{O and} >2)}{n(>2)} = \frac{3}{6} = \frac{1}{2}$

They are equally likely.

5. A survey was done of commuters in three major cities about how they primarily got to work. The results are shown in the frequency table below. Answer the following conditional probability questions.

	Car	Train	Walk	Total
New York	.05	.25	.10	.40
Los Angeles	.18	.12	.05	.35
Chicago	.08	.14	.03	.25
Total	.31	.51	.18	1.00

(a) What is the probability that a person picked at random would take a train to work given that they live in Los Angeles.

$P(\text{train} \mid \text{LA})$

$$= \frac{P(\text{train and LA})}{P(\text{LA})} = \frac{0.12}{0.35} \approx 0.34$$

(b) What is the probability that a person picked at random would live in New York given that they drive a car to work.

$$P(\text{NYC} \mid \text{Car}) = \frac{P(\text{NYC and Car})}{P(\text{Car})} = \frac{0.05}{0.31} \approx 0.16$$

(c) Is it more likely that a person who takes a train to work lives in Chicago or more likely that a person who lives in Chicago will take a train to work. Support your work using conditional probabilities.

$$P(\text{Chi}|\text{train}) = \frac{P(\text{Chi and train})}{P(\text{train})} = \frac{0.14}{0.51} \approx 0.27$$

$$P(\text{train}|\text{Chi}) = \frac{P(\text{Chi and train})}{P(\text{Chi})} = \frac{0.14}{0.25} = 0.56$$

It is almost twice as likely that a person who lives in Chicago take a train to work than a person who takes a train to work lives in Chicago.

REASONING

6. The formula for conditional probability is: $P(\text{B} \mid \text{A}) = \dfrac{P(\text{A and B})}{P(\text{A})}$. Solve this formula for $P(\text{A and B})$.

$$P(\text{B}|\text{A}) = \frac{P(\text{A and B})}{P(\text{A})} \Rightarrow P(\text{A and B}) = P(\text{A}) \cdot P(\text{B}|\text{A})$$

7. We say **two events**, A and B, **are independent** if the following is true:

$$P(\text{B} \mid \text{A}) = P(\text{B}) \text{ and likewise } P(\text{A} \mid \text{B}) = P(\text{A})$$

Interpret what the definition of **independent events** means in your own words.

Let's just look at the first one: $P(\text{B}|\text{A}) = P(\text{B})$. What this equation says is that given that A has occurred, it does not change the overall probability that B will happen. So the probability B will happen given A has happened is just the overall probability of B happening.

In the previous lesson's homework we saw how the occurrence of one event could change the probability of another event. When this happens, we say the two events are **not independent** of one another. When the occurrence of one event has no effect on the probability of another event happening, we say the events are **independent**.

***Exercise* #1:** Classify each of the following scenarios as having events that are dependent or events that are independent.

(a) A person pulls a red marble out of a bag that has 5 blue and 7 red marbles and does not replace it. Then a person pulls another red marble. Is the probability of pulling the second red marble out dependent on pulling the first red marble? Explain.

$\frac{7}{12}$ 1st

$\frac{6}{11}$ 2nd

what you want / everything

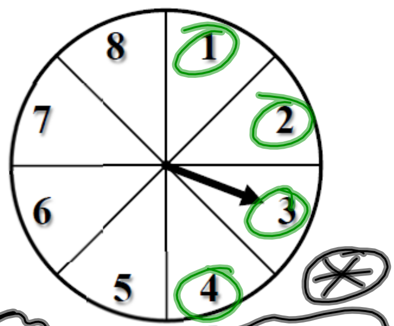
dependent (Probability changed)

(b) A person flips a coin and notes that it comes up heads. Then the person rolls a standard six-sided die and notes that it comes up as a number less than three. Is the probability that the number came up less than three dependent on getting a head when flipping the coin? Explain.

Independent coin does not effect the roll of the die.

The idea of **independence** is one that comes fairly naturally, but is important in order to see if there are associations amongst two events. Let's develop a tool to test dependence.

***Exercise* #2:** The spinner below is spun once and its outcome is noted. Let E be the event of getting an even, let P be the event of getting a prime, and let L be the event of getting a number less than 5. Find the following probabilities:



Probability stays same Independent
Probability changes dependent

(a) The probability of getting an even, i.e. $P(\mathrm{E})$.

$\frac{4}{8}$ or $\frac{1}{2}$

(b) The probability of getting an even given that the outcome was a prime number, i.e. $P(\mathrm{E} \mid \mathrm{P})$

4 prime 1 of which is even

$\frac{1}{4}$

(c) The probability of getting an even given that the outcome was a number less than 5, i.e. $P(\mathrm{E} \mid \mathrm{L})$

4 numbers less than 5 of which 2 are even

$\frac{2}{4}$ or $\frac{1}{2}$

(d) Which event does E depend on, P or L? How can you tell? What is a reasonable test?

E depends on P because the probability changed

DEFINITION OF INDEPENDENT EVENTS

Two events, A and B, are defined to be independent if:

$$P(A \mid B) = P(A) \quad \text{and likewise} \quad P(B \mid A) = P(B)$$

If probability changes then the events are dependent.

Exercise #3: A survey of 57 sixth graders was done to determine which subject was their favorite. The results are shown in the table below sorted by gender.

	Math	English	Social Studies	Science	Total
Female	8	6	10	6	30
Male	10	4	9	4	27
Total	18	10	19	10	57

(a) Does it appear, based on the data in this table, that the preference for math as a favorite subject has dependence on a student's gender? Show the analysis and explain your findings.

$P(\text{Math}) = \frac{18}{57} = 32\%$

$P(\text{Math} \mid \text{Male}) = \frac{10}{27} = 37\%$

Different therefore Dependent.

(b) Does it appear, based on the data in this table, that the preference for social studies as a favorite subject has dependence on a student's gender? Show the analysis and explain your findings.

$P(\text{Social Studies}) = \frac{19}{57} = 33\%$

$P(\text{SS} \mid \text{Female}) = \frac{10}{30} = 33\%$

Same, therefore Independent

There is a nice test for dependence that can be applied easily and comes from our formula for conditional probability from the last lesson.

Exercise #4: Given that $P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$, do the following.

(a) If A and B are independent, then rewrite this formula and solve for $P(A \text{ and } B)$.

$P(A \mid B) = P(A)$

$P(B) \cdot P(A) = \frac{P(A \text{ and } B)}{\cancel{P(B)}} \cdot \cancel{P(B)}$

$\boxed{P(B) \cdot P(A) = P(A \text{ and } B)}$ ← If this is true then A and B are independent

(b) The probability that a person is left handed is 12%, the probability they have brown eyes is 42% and the probability they have brown eyes and are left handed is 2%. Is the event of having brown eyes independent of being left handed? Support your answer.

$(.12)(.42) = .02$

$.0504 \neq .02$

Dependent

THE PRODUCT TEST FOR INDEPENDENCE